Exercise 4.8 b) Give an explicit solution.

Minimizing a linear function over a halfspace.

\[
\begin{align*}
\text{minimize} & \quad c^T x \\
\text{subject to} & \quad a^T x \leq b
\end{align*}
\]

where \( a \neq 0 \).

Exercise 4.11

Formulate the following problem as an LP. Explain in detail the relation between the optimal solution of the problem and the solution of its equivalent LP.

Minimize \( \|Ax - b\|_\infty \)

Exercise 4.12

Network flow problem. Consider a network of \( n \) nodes, with directed links connecting each pair of nodes. The variables in the problem are the flows on each link: \( x_{ij} \) will denote the flow from node \( i \) to node \( j \). The cost of the flow along the link from node \( i \) to node \( j \) is given by \( c_{ij} x_{ij} \), where \( c_{ij} \) are given constants. The total cost across the network is

\[
C = \sum_{i,j=1}^{n} c_{ij} x_{ij}
\]

Each link flow \( x_{ij} \) is also subject to a given lower bound \( l_{ij} \) (usually assumed to be nonnegative) and an upper bound \( u_{ij} \). The external supply at node \( i \) is given by \( b_i \), where \( b_i > 0 \) means an external flow enters the network at node \( i \), and \( b_i < 0 \) means that at node \( i \), an amount \( |b_i| \) flows out of the network. We assume that \( 1^T b = 0 \), i.e., the total external supply equals total external demand. At each node we have conservation of flow: the total flow into node \( i \) along links and the external supply, minus the total flow out along the links, equals zero. The problem is to minimize the total cost of flow through the network, subject to the constraints described above. Formulate this problem as an LP.

Exercise 4.21 a) Give an explicit solution

Minimizing a linear function over an ellipsoid centered at the origin

\[
\begin{align*}
\text{minimize} & \quad c^T x \\
\text{subject to} & \quad x^T A x \leq 1
\end{align*}
\]

where \( A \in S_{++}^n \) and \( c \neq 0 \).
Exercise 4.23 Formulate the $\ell_4$-norm approximation problem as a QCQP.

minimize $\|Ax - b\|_4 = (\sum_{i=1}^{m} (a_i^T x - b_i)^4)^{1/4}$

The matrix $A \in \mathbb{R}^{m \times n}$ (with rows $a_i^T$) and the vector $b \in \mathbb{R}^m$ are given.